Class: 9th

HERON'S FORMULA

Q1:→ A traffic signal board including school ahead is an equilateral triangle with side 'a'. find the area of the signal board using Heron's formula. If its perimeter is 180 cm. what will be the area of the signal board.

Ans:→ Each side of triangle = a

Perimeter of triangle = 3a

but given perimeter = 180 cm

$\therefore\ 3a = 180\,cm$

$\Rightarrow a = 60\,cm \rightarrow a = b = c = 60\,cm$

Now Semi perimeter (S) $= \dfrac{a+b+c}{2} = \dfrac{60+60+60}{2} = \dfrac{180}{2}$

$= 90\,cm$

$\therefore$ Area of Δ by Heron's formula $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{90(90-60)(90-60)(90-60)}$

$= \sqrt{90 \times 30 \times 30 \times 30}$

$= \sqrt{3 \times \underbrace{30 \times 30} \times \underbrace{30 \times 30}}$

$= 30 \times 30\sqrt{3}$

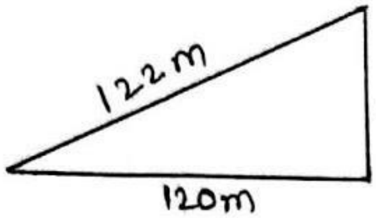
$= 900\sqrt{3}\,cm^2$ Ans.

Q2:→ The triangular wall of flyover - - - - How much did it pay?

Sol:→ Here $a = 122m$, $b = 120m$, $c = 22m$

$S = \dfrac{a+b+c}{2} = \dfrac{122m + 120m + 22m}{2} = \dfrac{264}{2} = 132m$

$\therefore$ Area of the triangular side wall $= \sqrt{s(s-a)(s-b)(s-c)}$



$= \sqrt{132(132-122)(132-120)(132-22)}$

$= \sqrt{132 \times 10 \times 12 \times 110}\ m^2$

$= 1320\ m^2$

Rent of one metre of wall for 1 year = Rs 5000

" " " " 1 month $= \frac{5000}{12}$

" " " " 3 month $= \frac{5000}{12} \times 3$

Rent of $1320\ m^2$ of wall for 3 moths $= \frac{5000}{12} \times 3 \times 1320$

= Rs 16 50,000 Ans//

Q3 :→ There is a slide in a park ---- find the area painted in Colour.

Sol :→ Here $a = 15m$, $b = 11m$, $C = 6m$

$$S = \frac{a+b+c}{2} = \frac{15+11+6}{2}\ m = \frac{32}{2} = 16\ m$$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16 \times 1 \times 5 \times 10}$

$= \sqrt{\underbrace{4 \times 4} \times 1 \times \underbrace{5 \times 2 \times 5}}$

$= 4 \times 5\sqrt{2} = 20\sqrt{2}\ m^2$ Ans//

Q4 ⟹ Find the area of triangle two side of which are 18cm and 10cm and the perimeter is 42cm.

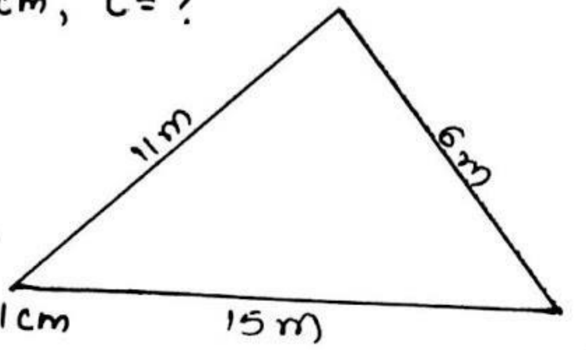
Sol: $\longrightarrow$ Here $a = 18cm$, $b = 10cm$, $c = ?$

Perimeter $= 42cm$

$\Rightarrow a+b+c = 42\,cm$

$\Rightarrow 18+10+c = 42\,cm$

$\Rightarrow c = 42-18-10 = 21\,cm$



Now $s = \dfrac{a+b+c}{2} = \dfrac{18+10+14}{2} = \dfrac{42}{2} = 21\,cm$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}\ cm^2$

$= \sqrt{3 \times 7 \times 3 \times 11 \times 7}$

$= 3 \times 7\sqrt{11}\ cm^2$

$= 21\sqrt{11}\ cm^2$

Q5 $\Longrightarrow$ Side of a triangle are in ratio 12:17:25 and its perimeter is 540cm. Find its area

Sol: $\longrightarrow$ Let the sides of triangle be $12x$, $17x$ and $25x$

$\therefore 12x + 17x + 25x = 540$

$\Rightarrow 54x = 540$

$\Rightarrow x = \dfrac{540}{54} = 10$

$\therefore a = 12x = 12 \times 10 = 120\,cm$

$b = 17x = 17 \times 10 = 170\,cm$

$c = 25x = 25 \times 10 = 250\,cm$

Now $s = \dfrac{120+170+250}{2} = \dfrac{540}{2} = 270\,cm$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}$

$= \sqrt{270 \times 150 \times 100 \times 20} \quad cm^2$

$= \sqrt{81000000} = 9000 cm^2$

Q6 :→ An isosceles triangle has perimeter 30 cm and each of equal sides is 12 cm. Find the area of triangle

Sol :→ Here $a = b = 12cm$

also $a + b + c = 30$ cm

$\therefore 12 + 12 + c = 30$ cm

$c = 30 - 12 - 12 = 6$ cm

$\therefore S = \frac{12+12+6}{2} = \frac{30}{2} = 15cm$

Now Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15 \times 3 \times 3 \times 9}$

$= \sqrt{\underbrace{3 \times 5 \times 3}\times \underbrace{3 \times 3} \times 3}$

$= 3 \times 3 \sqrt{5 \times 3}$

$= 9\sqrt{15}\ cm^2$ Ans//

Exercise 12.2

Q1 A park in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$. $AB = 9m$, $BC = 12m$, $CD = 5m$, $AD = 8m$, How much area does it occupy?

Sol :→ As ABCD is a park

Join BD

Now In ΔDBC we have $\angle C = 90^\circ$

$\therefore \quad DB^2 = BC^2 + CD^2$ (By pyth theorem)

$\Rightarrow DB^2 = (12)^2 + 5^2$

$\Rightarrow DB = \sqrt{144 + 25} = \sqrt{169}$

$\Rightarrow DB = 13m$

$\therefore$ Area of $\Delta DBC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{\cancel{2}} \times \overset{6}{\cancel{12}} \times 5 = 30m^2$

In ΔABD $a = 9m$, $b = 8m$, $C = 13m$

$\therefore \quad S = \frac{a+b+c}{2} = \frac{9+8+13}{2} = 15m$

$\therefore$ Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-9)(15-8)(15-13)} \; m^2$

$= \sqrt{15 \times 6 \times 7 \times 2} \; m^2$

$= \sqrt{1260} = 35.5m^2$ (Approx.)

$\therefore$ Area of park $= (30 + 35.5) = 65.5\ m^2$ Ans.

Q2 Find the area of quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm

Sol: $\longrightarrow$ In ΔABC we have

$$AB^2 + BC^2 = AC^2$$

$$(3)^2 + (4)^2 = AC^2$$

$$\Rightarrow 9 + 16 = (5)^2$$

$$\Rightarrow 25 = 25$$

Hence By inverse of Pythagoras theorem

ΔABC is a right angled triangle

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{Alt}$

$$= \frac{1}{\cancel{2}} \times 3 \times \cancel{4}\ 2$$

$$= 6\ cm^2$$

In ΔACD, $a = 5$ cm, $b = 4$ cm $c = 5$ cm

$$\therefore S = \frac{5+4+5}{2} = \frac{14}{2} = 7\text{cm}$$

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{7\,(7-5)(7-4)(7-5)}$$

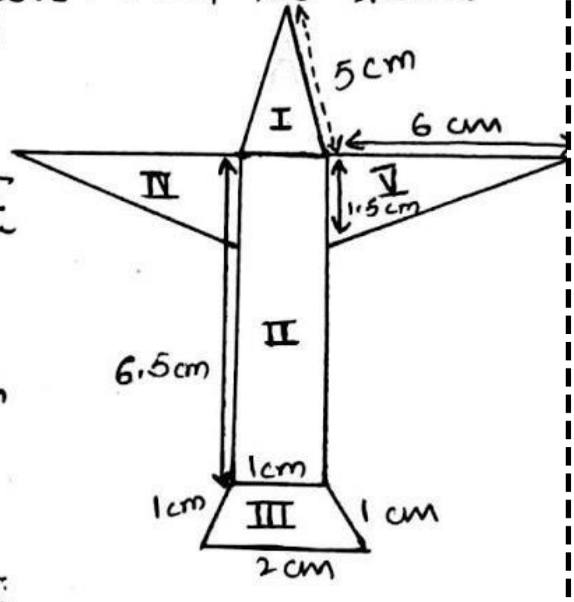
$$= \sqrt{7 \times 2 \times 3 \times 2}$$

$$= \sqrt{84}\ cm^2 = 9.2\ cm^2 \text{ (approx.)}$$

$\therefore$ Area of quad $= (6 + 9.2)$ cm $= 15.2\ cm^2$ Ans.

(4)

Q3 Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used



Sol :→ For the triangle marked I

$a = 5\,cm,\ b = 5\,cm \quad c = 1\,cm$

$\therefore\ S = \frac{5+5+1}{2} = \frac{11}{2} = 5.5\,cm$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}$

$= \sqrt{6.1875}\ cm^2 = 2.5\,cm^2$

For the rectangle marked II :→

Length $= 6.5\,cm$

breadth $= 1\,cm$

$\therefore$ Area of rectangle $= L \times b$

$= 6.5 \times 1\,cm = 6.5\,cm^2$

For the terpezium marked III :→

Draw $AF \parallel DC$ and $AE \perp BC$

$AD = FC = 1\,cm, \quad DC = AF = 1\,cm$

$\therefore\ BF = BC - FC = (2-1) = 1\,cm$

Hence ΔABF is equilateral

Also, E is mid-point of BF

$\therefore BE = \frac{1}{2}cm = 0.5 cm$

$AB^2 = AE^2 + BE^2$ (By Pythagoras theorem)

$\Rightarrow AE^2 = 1^2 - (0.5)^2 = 0.75$

$AE = 0.9 cm$ (aprox)

$\therefore$ Area of terpezium $= \frac{1}{2}$ (Sum of llel sides) $\times$ height

$= \frac{1}{2} \times (BC + AD) \times AE$

$= \frac{1}{2} \times (2+1) \times 0.9$

$= \frac{1}{2} \times 3 \times \frac{9}{10}$

$= 1.4 cm^2$

For the triangle marked IV : $\rightarrow$

It is a right-triangle

$\therefore$ Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 6 \times 1.5 = 4.5 cm^2$

For the triangle marked V: $\rightarrow$

This triangle is congurent to the triangle marked IV

Hence area of triangle V $= 4.5 cm^2$

$\therefore$ Total area of paper used $= (2.5 + 6.5 + 1.4 + 4.5 + 4.5)$

$= 19.4 cm^2$ Ans

Q4 : $\rightarrow$ A triangle and parallelogram ------ find the height of the parallelogram

Sol:→ In the figure ABCD is a ||gram and ABE is the triangle which stands on Base AB

For ΔABE $a = 30cm$, $b = 28cm$

$c = 26cm$

$\therefore S = \frac{a+b+c}{2} = \frac{30+28+26}{2} = \frac{84}{2} = 42cm$

$\therefore$ Area of $\Delta ABE = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-30)(42-28)(42-26)}$

$= \sqrt{42 \times 12 \times 14 \times 16}$

$= \sqrt{112896}\ cm^2$

$= 336\ cm^2$

Now area of ||gram = base × height

$\Rightarrow 336 = 28 \times$ height

$\Rightarrow$ height $= \frac{336}{28} = 12cm$ Ans//

Q5 ⟹ A rhombus shaped field ---- area of grass field which each cow be getting?

Sol:→ Clearly the diagonal AC of the rhombus divides it into two congurent triangles

For ΔABC

$a = b = 30m$, $c = 48m$

$\therefore S = \frac{a+b+c}{2} = \frac{30+30+48}{2}$

$= 54m$

Now area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-30)(54-30)(54-48)}$

$= \sqrt{54 \times 24 \times 24 \times 6} = 432\,m^2$

$\Rightarrow$ Area of rhombus $= 2 \times 432\,m^2$

$= 864\,m^2$

Number of cows $= 18$

Hence area of grass field which each cow gets $= \frac{864}{18}\,m^2 = 48\,m^2$ Ans

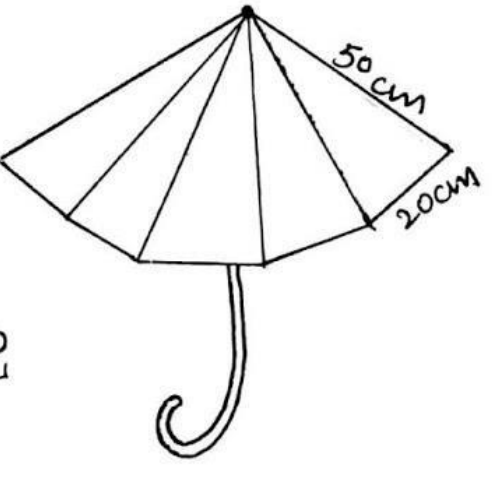
Q6 An Umbrella is made by - - - - required for the Umbrella ?

Sol :→ firstly we find area of triangular piece

Here $a = b = 50cm$, $c = 20cm$

$\therefore S = \frac{a+b+c}{2} = \frac{50+50+20}{2} = \frac{120}{2} = 60m$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$



$= \sqrt{60(60-50)(60-50)(60-20)}$

$= \sqrt{60 \times 10 \times 10 \times 40}$

$= \sqrt{\underbrace{6 \times 10 \times 10}_{} \times \underbrace{10 \times 2 \times 2 \times 10}_{}}$

$= 200\sqrt{6}\ cm^2$

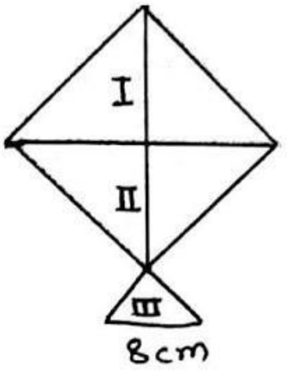
Area of 10 such pieces $= 10 \times 200\sqrt{6} = 2000\sqrt{6}\ cm^2$

Hence cloth required for each colour $= \frac{2000\sqrt{6}}{2}$

$= 1000\sqrt{6}\ cm^2$ Ans

Q7 A kite is in the shape --- each shade has been used in it?

Sol: $\longrightarrow$



ABCD is a square

So $AO = OC$, $OB = OD$

and $\angle AOB = 90^\circ$ {Diagonal of a square bisect each other at right angles}

$BD = 32\text{ cm}$ Given

$\Rightarrow OA = \frac{32}{2} = 16\text{ cm}$

, ΔABD is a right triangle

So, area of $\Delta ABD = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 32 \times 16 = 256\text{ cm}^2$

Thus area of triangle $\Delta BCD = 256\text{ cm}^2$

For triangle CEF, $a = b = 6\text{ cm}$, $c = 8\text{ cm}$

$\therefore\ S = \frac{a+b+c}{2} = \frac{6+6+8}{2}\text{ cm} = 10\text{ cm}$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-6)(10-6)(10-8)}$

$= \sqrt{10 \times 4 \times 4 \times 2} = \sqrt{320}\text{ cm}^2$

$= 17.92\text{ cm}^2$

Hence paper needed for shade $I = 256\text{ cm}^2$ for shade $II = 256\text{ cm}^2$ and for shade $III = 17.92\text{ cm}^2$

Q8: $\longrightarrow$ A floral design on a floor is ----- polishing the tiles at the rate of 50 p per cm^2

Sol :→ we have lengths of the sides of trangular tiles are

$a = 35 cm,\ b = 28 cm,\ c = 9 cm$

$\therefore\ s = \frac{a+b+c}{2} = \frac{35+28+9}{2}$

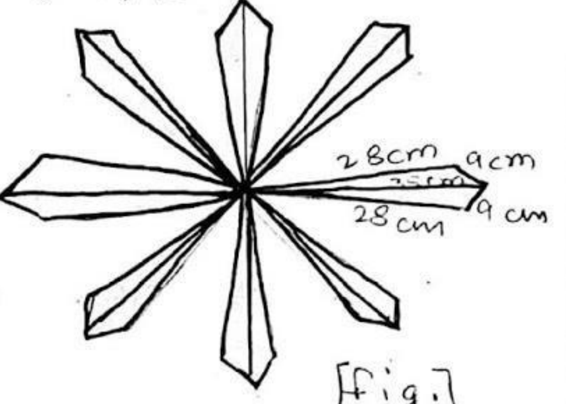
$= \frac{72}{2} = 36 cm$

$\therefore$ Area of 1 triangular tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36(36-35)(36-28)(36-9)}$

$= \sqrt{36 \times 1 \times 8 \times 27}$

$= \sqrt{7776}\ cm^2 = 88.2\ cm^2$



[fig.]

$\therefore$ Area of 16 such tiles $= 16 \times 88.2\ cm^2$

cost of polishing $1 cm^2 = 50p = Rs\ 0.50$

$\therefore$ Total cost of polishing the floral design

$= 16 \times 88.2 \times 0.50$

$=$ Rs 705.60 Ans//

Q9 :→ A field is in the shape of terpezium - - - - area of the field.

Sol :→ In the figure ABCD is a field

DC = AF = 10 m

AD = FC = 13 m

For Δ BCF, a = 15m, b = 14m c = 13m

$\therefore\ s = \frac{15+14+13}{2} = \frac{\cancel{42}}{\cancel{2}} = 21 m$

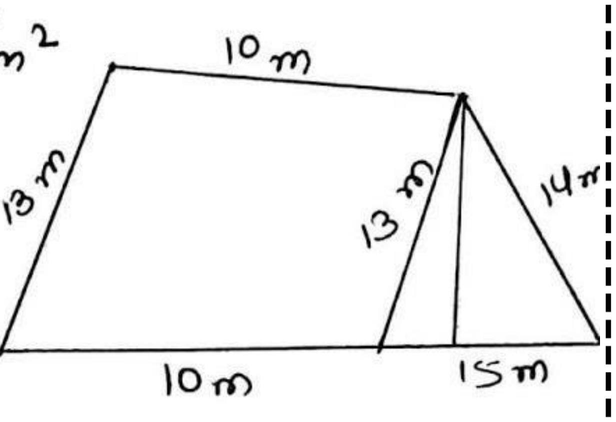
$\therefore$ Area of $\Delta BCF = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-14)(21-13)}\ m^2$

$= \sqrt{21 \times 6 \times 7 \times 8}\ m^2$

$= \sqrt{7056}\ m^2$

$= 84\ m^2$



Also area of $\Delta BCF = \frac{1}{2}\ b \times A$

$= \frac{1}{2} \times BF \times CG$

$\Rightarrow 84 = \frac{1}{2} \times 15 \times CG$

$\Rightarrow CG = \frac{84 \times 2}{15} = 11{\cdot}2\ m$

$\therefore$ Area of terpezium $= \frac{1}{2}$ (sum of $\parallel$el sides) $\times$ Alt

$= \frac{1}{2} \times (25+10) \times 11{\cdot}2\ m^2$

$= 196\ m^2$

Hence area of the field $= 196\ m^2$ Ans,,